

Random (Un)rounding : Vulnerabilities in Discrete Attribute Disclosure in the 2021 Canadian Census

CHRIS WEST, IVY VECNA, and RAIYAN CHOWDHURY, University of Waterloo, Canada

The 2021 Canadian census is notable for using a unique form of privacy, random rounding, which independently and probabilistically rounds discrete numerical attribute values. In this work, we explore how hierarchical summative correlation between discrete variables allows for both probabilistic and exact solutions to attribute values in the 2021 Canadian Census disclosure. We demonstrate that, in some cases, it is possible to "unround" and extract the original private values before rounding, both in the presence and absence of provided population invariants. Using these methods, we expose the exact value of 624 previously private attributes in the 2021 Canadian census disclosure. We also infer the potential values of more than 1000 private attributes with a high probability of correctness. Finally, we propose how a simple solution based on unbounded discrete noise can effectively negate exact unrounding while maintaining high utility in the final product.

CCS Concepts: • **Random Rounding**; • **Census of Population**; • **Re-identification**;

1 INTRODUCTION

Collecting a census of population is an important undertaking. Uses of census data include planning services such as schools and childcare, defining voting districts, and modeling diseases such as COVID-19 [8]. However, the collection and publication of data should not be taken lightly. Even when only aggregate statistics are published, individual personal information can sometimes be recovered [12]. Thus, when releasing such statistics, one must use some mechanism to protect the privacy of individuals represented within the dataset. This is not only a matter of ethics; it is also a matter of policy, as the release of data may be subject to various regulations. Statistics Canada, which collects and disseminates the Canadian census data, has legal obligations from the Statistics Act [7] and the Privacy Act [6] to protect the privacy of data subjects. Similarly, the United States under Title 13 protects against the disclosure of personally-identifiable census data with both severe monetary penalties and potential prison time for a failure to do so [5].

The United States Census Bureau used a form of differential privacy to protect the published results from its 2020 census [2]. Differential privacy provides a formal, mathematically provable privacy guarantee. The main privacy protection used in the most recent Canadian census is random rounding [8], where counts are randomly rounded either up or down to a multiple of 5. Counts closer to the next multiple of 5 have a higher probability of rounding up, and counts closer to the previous multiple of 5 have a higher probability of rounding down. This process is formally defined in Section 2.1.

Random rounding provides 1) utility in that we are certain of the range of the true attribute value, as well as 2) privacy in that the true value is effectively hidden within this range. However, as we will show, two key choices in implementation for the 2021 Canadian census disclosure expose significant vulnerabilities in the random rounding process. These choices are 1) treating correlated attribute values as independent in the context of random rounding, as well as 2) publishing population-wide invariants which significantly narrow the scope of enumeration.

Using publicly available Canadian census data [3], we identify the following contributions from our work:

- (1) We show that hundreds of exact attribute values can be extracted from correlations in rounded data in the presence of population invariants.

- (2) We hypothesize and test a similar method that can be used to extract exact attribute values from rounded data that does not require population invariants.
- (3) We demonstrate a way to extract probabilistic information about randomly rounded attributes, with and without invariants, and use this method to predict the true value of more than 1000 attributes with high confidence.
- (4) We propose a simple alternative to random rounding which both increases net utility while mitigating the potential for exact inference.

Although we did not expose individual-level personal information, the inference of randomly-rounded attributes implies a weakness in the mechanism and the potential for further exploitation. It is our hope that this work motivates the adoption of more rigorous privacy protections in future census collections and distributions.

2 BACKGROUND

2.1 Random Rounding

Canada's 2021 Census of Population uses two main methods to protect the privacy of its subjects: suppression of certain results in areas with fewer than 40 people (or 100 in some cases) and random rounding of counts [8]. Statistics Canada kindly clarified the random rounding algorithm for us: All counts are rounded to a nearby multiple of 5. Note that this is not to the nearest multiple, but instead follows a probabilistic pattern. The random rounding algorithm $rround$ on some non-negative integer x can be succinctly expressed as follows:

Definition 2.1. $rround$.

$$rround(x) = \begin{cases} x - x \bmod 5 & \text{with probability } 1 - \frac{x \bmod 5}{5} \\ x - x \bmod 5 + 5 & \text{with probability } \frac{x \bmod 5}{5} \end{cases} \quad (1)$$

x	$\mathbf{P}[rround(x) = 10]$	$\mathbf{P}[rround(x) = 15]$	$\mathbf{P}[rround(x) = 20]$
10	1	0	0
11	4/5	1/5	0
12	3/5	2/5	0
13	2/5	3/5	0
14	1/5	4/5	0
15	0	1	0
16	0	4/5	1/5
17	0	3/5	2/5
18	0	2/5	3/5
19	0	1/5	4/5
20	0	0	1

Table 1. Outcome probabilities for random rounding on the integers 10-20.

Random rounding gives some form of plausible deniability. For instance, a published value of 15 implies that the true number lies anywhere in the range [11,19], with a higher probability density around the center of 15. This also gives obvious utility; we can say with certainty that the published value is no more than 4 away from the true value in any direction. This may be a large amount for small census subdivisions, but at the town, city, or province level, it can become quite a strong statement.

However, an important aspect to note is that all attribute values are rounded *individually*, even those which are correlated. For example, if a region publishes the rounded attributes:

- A1: Those who speak only one of English or French
- A2: Those who speak only English
- A3: Those who speak only French

The published values will be rounded independently, despite the fact that $A1_{True} = A2_{True} + A3_{True}$. We will soon demonstrate that this is a significant design flaw.

As a side note, we have chosen to look at attribute values only greater than 10. This is because random rounding is not as well defined in the range [0-9]. Our preliminary experiments showed inconsistency in random rounding for integers less than 10, and some documentation suggests that a mod-10 rounding scheme is used below 10 (although Statistics Canada was not clear on this point). For simplicity, we stick to larger integers that are well-understood and well-defined. Although this limits the scope of experiments, it is more than sufficient for demonstrating significant vulnerabilities.

2.2 Invariants

An interesting choice for the 2021 Canadian census disclosure was to retain net population *invariants*. Invariants are exact values that are not subject to the randomization or noising process and are retained in the final census disclosure. For example, the city of Thunder Bay has a listed population of 108843 (rather than 108840 or 108845, as would be given by random rounding). Note that the United States 2020 census also chose to include certain invariants, such as state-level population and block-level housing unit counts [4]. There can be a number of reasons to include invariants in a census disclosure. They provide obvious utility, and it may be necessary to have an exact value for policy or legal reasons. In the case of the US census, a major reason for having a state-level population invariant is to provide concrete figures for reapportionment of congressional seats [13]. However, we will soon show that the inclusion of even the single net population invariant can have significant consequences for otherwise protected randomly-rounded attributes.

2.3 SAT and SMT Solvers

SAT and SMT solvers are a general class of algorithms that are designed to solve satisfiability problems. The most common of these problems is that of the Boolean Satisfiability Problem (SAT), which seeks to find candidate assignments of Boolean variables to satisfy logical statements. However, modern solvers are highly flexible and efficient and thus can solve complex integer programming problems subject to well-defined constraints.

SAT solvers have previously been used to great success for database reconstruction attacks. For instance, SAT solvers were used for generating candidate solutions to an example census region as a proof of concept for demonstrating the importance of proper privacy measures in census disclosures [11]. Internal experiments on 2010 US census and redistricting data have shown that large-scale linear solvers can come up with candidate solutions, which in some cases may be exact or else share common attributes between them that allow for inference [1].

3 INFERENCE

3.1 Exact Inference

In the context of census attribute disclosure, exact inference means that we can generate a candidate solution to our system of variables before provably showing that this is the only possible solution that satisfies the constraints of the problem. We will demonstrate two different types of exact

inference methods; one that is reliant on the presence of invariants, as well as one that has no such requirement.

3.1.1 Invariant-based Exact Inference. To see why invariants pose a risk to random rounding attribute inference, it is sufficient to show an example.

We noted earlier that correlated attributes are rounded independently. Let us take the example of the discrete age histograms used at the start of each region block census disclosure. A simplified view of the published age variables is given as:

A0: Overall Net Population (Invariant)

- A1: Age 0-14
 - A2 Age 0-4
 - A3 Age 5-9
 - A4 Age 10-14
- A5: Age 15-64
- A6: Age 65+

Notice the hierarchical correlations between variables. Ages 0-14, 15-64, and 65+ make up an exact subset of the total population. We also know that the variables A2-A4 make up an exact cover and partition of A1. Therefore, we can design a system of variables such that:

$$A0_{True} = A1_{True} + A5_{True} + A6_{True}$$

$$A1_{True} = A2_{True} + A3_{True} + A4_{True}$$

Seeing as A0 is an invariant, $A0_{True} = A0$. However, A1-A6 are subject to random rounding so they may not take on the same value as the ground truth.

Now, let us consider two edge cases in which exact inference is uniquely possible. For simplicity, we will only consider A0, A1, A5, and A6.

	Real	Randomly Rounded
A0: Net Population	48 (Invariant)	48 (Invariant)
A1: Age 0-14	16	20
A5: Age 15-64	16	20
A6: Age 65+	16	20

Table 2. Sample Instance for Random Rounding #1

	Real	Randomly Rounded
A0: Net Population	72 (Invariant)	72 (Invariant)
A1: Age 0-14	24	20
A5: Age 15-64	24	20
A6: Age 65+	24	20

Table 3. Sample Instance for Random Rounding #2

Following our previous formulation, we know that $A0_{True} = A1_{True} + A5_{True} + A6_{True}$. However, by the bounds of random rounding, we know that:

$$\begin{aligned}
 16 &\leq A1_{True} \leq 24 \\
 16 &\leq A5_{True} \leq 24 \\
 16 &\leq A6_{True} \leq 24
 \end{aligned}$$

As it turns out, when all three attributes lie on the edge-case bound, there is only one viable solution. For instance, in Table 2, we can see quite naturally that the only possible solution is that $A1 = A5 = A6 = 16$ since there is no possible smaller random rounding of 3 variables from 20 which sums to 48. Likewise, for Table 3, similar logic holds that 24 is the maximum true value for all 3 variables given the published value of 20, and thus $A1 = A5 = A6 = 24$ is the only possible solution.

In fact, to find these kinds of vulnerabilities, it is sufficient to check if:

Let n = Number of correlated sub-attributes, and
 Corr = The list of correlated attributes

$$\sum_{i=1}^n A_{Corr[i]} = Invariant \pm 4n$$

Keep in mind that it is not necessary that all attributes are equal as we have shown in the examples so far (this was done only for the sake of simplicity). What is important is that all attributes fall on the same bound, whether that be upper or lower, before rounding to the opposite bound.

So far we have only demonstrated one possible use of the invariant for age attributes. However, the same logic also holds for sex(+)-based attributes. As shown in Fig 1, nearly every attribute in the Canadian census also has the corresponding Men(+) / Women(+) values (the 2021 Canadian census chose to use a binary division for sex and gender, so non-binary respondents are partitioned

CHARACTERISTIC_NAME	CHARACTERIST	C1_COUNT_TOTAL	SYMBOL	C2_COUNT_MEN+	SYMBOL_1	C3_COUNT_WOMEN+
Population, 2021		Invariant 40232				
Population, 2016	1	35874				
Population percentage change, 2016 to 2021		12.1				
Total private dwellings	2	19610				
Private dwellings occupied by i	3	17181				
Population density per square kilometre		0.1				
Land area in square kilometres		472345.44				
Total - Age groups of the population - 100% data		40230		20100		20130
0 to 14 years		6820		3545		3275
0 to 4 years		2175		1090		1085
5 to 9 years		2375		1245		1130
10 to 14 years		2275		1205		1065
15 to 64 years		27360		13405		13960
15 to 19 years		2085		1085		995
20 to 24 years		2140		1080		1060
25 to 29 years		2705		1310		1390
30 to 34 years		3415		1665		1750
35 to 39 years		3425		1660		1770
40 to 44 years		2840		1365		1470
45 to 49 years		2615		1215		1400
50 to 54 years		2460		1220		1235
55 to 59 years		2805		1355		1440
60 to 64 years		2880		1430		1450
65 years and over		6050		3155		2890
65 to 69 years		2370		1225		1150
70 to 74 years		1765		925		840
75 to 79 years		1025		560		460
80 to 84 years		505		270		230
85 years and over		385		175		210
85 to 89 years		255		115		140
90 to 94 years		110		50		55
95 to 99 years		20		10		15
100 years and over		0		0		0

Fig. 1. Invariant-based Inference, with 1-2 Levels of Potential Compound Inferences Shown

into one of these two inclusive (+) categories). These are subject to the same independent random rounding rules as before and are not treated as invariants.

Solutions generated by exact inference have a worrying potential for further exploitation. Notice that by finding an exact solution to these child attributes, we now open up the possibility for subsequent invariant-based inference. For instance, by inferring the sex(+) invariants in the net population, we open up the possibility of doing the same 3-category age inference attack now not only on the original data but also on the Men(+) and Women(+) attributes for each of the exposes ages. Similarly, in the age example above, by inferring A1 (alongside A5 and A6), we can now attempt a similar attack on A2-A4. That being said, the odds of this type of compound attack succeeding are relatively low due to the unlikely rounding events involved. We did some preliminary testing to check for these compound attacks but did not investigate them thoroughly.

3.1.2 Invariant-free Exact Inference. Although invariant-based attacks can be powerful, the only invariant provided in the 2021 Canadian census is the population for each census subdivision (country, provinces, regions, cities, and so on). This means that the scope of our attacks is limited only to those attributes which make up an exact subset of the total population of that subdivision. However, we can show that invariants are not always necessary to do a similar attack to the one shown earlier. Let us demonstrate with an example:

Consider the Celtic Languages category (mother tongue, single response):

- A581: Celtic Languages
 - A582: Irish
 - A583: Scottish Gaelic
 - A584: Welsh
 - A585: Celtic Languages, n.i.e.

As before, we require that the 4 sub-attributes are an exact subset of the parent attribute and that each attribute is independently randomly rounded. Since this is a single response category and n.i.e includes those Celtic languages "not included elsewhere", we can safely assume that this is an exact and mutually exclusive subset. We have included what we believe to be all or at least most of said categories, 20 in total, in Table 11.

Unlike in previous examples, we are not given the parent attribute as an invariant; instead, it is also subject to random rounding. Now, let us show two sample instances in tables 4 and 5.

Notice in these two instances that, although we do not have an exact value for the parent attribute, we have a range. For instance, in Table 4, we know that $56 \leq A581_{True} \leq 64$. However, we also know that:

$$\begin{aligned} 16 &\leq A582_{True} \leq 24 \\ 16 &\leq A583_{True} \leq 24 \\ 16 &\leq A584_{True} \leq 24 \\ 16 &\leq A585_{True} \leq 24 \end{aligned}$$

Since $16 \cdot 4 = 64$ is the assignment that produces the minimal sum solution, we are certain that the only possible solution is:

$$\begin{aligned} A581_{True} &= 64 \\ A582_{True}, A583_{True}, A584_{True}, A585_{True} &= 16 \end{aligned}$$

	Real	Randomly Rounded
A581: Celtic Languages	64	60
A582: Irish	16	20
A583: Scottish Gaelic	16	20
A584: Welsh	16	20
A585: Celtic Languages, n.i.e.	16	20

Table 4. Sample Instance for Invariant-Free Inference

	Real	Randomly Rounded
A581: Celtic Languages	76	80
A582: Irish	19	15
A583: Scottish Gaelic	19	15
A584: Welsh	19	15
A585: Celtic Languages, n.i.e.	19	15

Table 5. Sample Instance for Invariant-Free Inference

Similar logic follows for the opposite case for the upper bound, but we will not show it for brevity.

In order to check for this vulnerability in a subset of the data, one only has to find attributes with 4 correlated sub-attributes as an exact and mutually exclusive subset, and then check if:

Let Corr = The list of correlated attributes

$$\sum_{i=1}^4 A_{Corr[i]} = A_{parent} \pm 20$$

Unlike in invariant-based exact inference, note that the number of sub-attributes is constrained. This is because we are invoking edge-case behavior in the summative attribute that only works with certain combinations of both upper and lower bounds. If we consider 2-3 sub-attributes, both the maximum and minimum of their bounds put us in the middle of the parent attribute probability distribution, which has non-deterministic rounding behavior. For instance, $6+6=12$, $9+9=18$, $6+6+6=18$, and $9+9+9=27$ all are uninteresting cases. As it turns out, groups of 4 sub-attributes with a single parent attribute turn out to be the most exciting and exploitable case.

It is theoretically possible to exploit the same vulnerability with 9,14,19 etc. attributes, but the unlikeliness of the rounding becomes vanishingly small with that many attributes. This is because it requires the independent rounding of those many attributes in the exact correct (and unlikely) direction for that attack to work. We will see in the results section how even the basic case with 4+1 attributes is statistically very unlikely.

3.2 Probabilistic Inference

The final type of inference which we will partially explore is probabilistic inference.

By probabilistic inference, we mean that we now reduce the scope of the problem such that instead of saying "we are certain that attribute A_i takes on value x ", we can instead say "we are 40% sure that attribute A_i takes on value x , 30% that it takes on value $x+1$, etc."

This is naturally a weaker attack, but it has a much more broad application due to the fact that it does not require very specific edge-case rounding behavior. As well, depending on the threat

model, a 30-50% certainty in the true value may be sufficient. Not only this, but a probabilistic model can very often rule out other candidate solutions.

Let us consider a small toy example where this may be interesting. If the total population is 87, and the posted male(+) population is 35 and the posted female(+) population is 45, we can not do exact inference. However, notice that there are only 2 possible solutions:

$$M(+) = 38, F(+) = 49 \text{ or}$$

$$M(+) = 39, F(+) = 48$$

This may still be an interesting result for an adversary, even in the absence of definitive results.

For our upcoming experiments, we will explore two different methods of doing probabilistic inference. One of these methods gives precise statistical information about arbitrary attribute value probability based on SAT solver enumeration but is in general not very practical for finding vulnerabilities in real data. The other is a simple logic-based method similar to those which we have used before. Both can be done with and without the presence of an invariant.

3.2.1 Invariant-based Probabilistic Inference. To determine a precise probability distribution for attribute values, it is possible to employ the use of an SMT solver to enumerate possibilities for solving the system of equations defined by the random rounding constraints. Using our example for age constraints from the previous section, we define something like:

$$\begin{aligned} A0 &= A1_{True} + A5_{True} + A6_{True} \\ A1 - 4 &\leq A1_{True} \leq A1 + 4 \\ A5 - 4 &\leq A5_{True} \leq A5 + 4 \\ A6 - 4 &\leq A6_{True} \leq A6 + 4 \end{aligned}$$

This is basically the same fundamental system as before so far. However, instead of looking for specific edge cases, we enumerate all possible cases and calculate their relative probabilities.

First, we generate a potential solution to the system of equations using the Python SMT solver Z3. [10] Note that the generated solution is only *a* feasible solution, but is not guaranteed to be the only feasible solution. We must enumerate all other solutions as well. To do this, we add on the current solution as a NOT constraint and rerun the code to generate another solution. This continues until the space of possible solutions is exhausted. Note that the solution space for a particular instance is not easy to express in closed form, and can easily be on the order of hundreds to even tens of thousands or more solutions, depending on the value of the invariant and the number of correlated variables. That being said, the method theoretically composes very well with additional information about the feature space, since this narrows down the search and enumeration space as well.

Once all solutions have been generated, it is not as simple as looking at common features between solutions since not all solutions are equally probable. Instead, we can observe a relative probability distribution of each solution by the odds of the observed random rounding occurring given that solution. For instance, [19 19 19] rounding to [15 15 15] is much less likely than [16 16 16] rounding to [15 15 15]. Since we have the exact odds of random rounding, we can get the exact odds of a possible rounding and then scale this to the total probability of all feasible solutions in a post-processing step. With this, we can rank the most likely solutions as well as their overall probability in the solution space. We will then be able to say something like "The most likely solution is [12, 26, 40] at 4% probability, and it is 5% more likely than the next most common solution. There are exactly 412 unique solutions."

This is clearly an interesting result but is not always particularly informative in and of itself. This is to say that the most likely solution may still only be vanishingly likely due to the size of the solution space. As well, we are more interested in the value or distribution of features rather than the total solution itself since features are more likely to reveal information about a single individual.

To address this, we extend our framework to examine feature values given the probability of their parent solution. Since the probability of the parent solution already takes into account the odds of a given random rounding, we only need to iterate through our solutions and construct the probability histogram for all possible solutions to each feature individually. This is what we will demonstrate on some sample census data in the results section.

This method gives precise information about all attributes in your system but is computationally costly. Since the focus of this paper is on looking for specific vulnerabilities in the Canadian 2021 census data, it is also possible to use a simple logic-based method to find non-exact invariant-based inferences. We can simply check for a condition like:

Let n = Number of correlated sub-attributes, and
Corr = The list of correlated attributes

$$\sum_{i=1}^n A_{Corr[i]} = (Invariant \pm (4n - 1))$$

Notice that this is almost identical to the previous condition for exact invariant-based inference, although it is slightly looser. The result of this is that we can infer the value of all but one of the attributes in the system, and thus can say that we are confident about the value of the attribute to a probability of $1 - \frac{1}{n}$ for $n \geq 2$. As we will explore later, this type of attack can still be relatively strong while also being much more likely than the exact-inference equivalent. It is also much less demanding computationally, and so is what we use in practice.

3.2.2 Invariant-free Probabilistic Inference. We will also demonstrate that we can do similar methods to the above without the need for an invariant. To show this, let us pretend we are analyzing one of the nearly *any* attributes which are non-invariant and have both a male(+) and female(+) component.

We will essentially format the same general system of equations as above, but we no longer have an equality equation to limit the system. Instead, our invariant only has a rough bound:

$$\begin{aligned} A0_{Male, True} + A0_{Female, True} &\geq A0 - 4 \\ A0_{Male, True} + A0_{Female, True} &\leq A0 + 4 \\ A0 - 4 &\leq A0_{True} \leq A0 + 4 \\ A0_{Male} - 4 &\leq A0_{Male, True} \leq A0_{Male} + 4 \\ A0_{Female} - 4 &\leq A0_{Female, True} \leq A0_{Female} + 4 \end{aligned}$$

As before, we can enumerate all possible solutions, rank them by the relative odds of that solution based on random rounding, and then extract individual feature probabilities. There will most likely be many more solutions than in the previous case since the invariant is non-fixed, but unlike the previous invariant-free exact inference technique, we can technically do this on any size of grouped features (albeit with limitations to due computational complexity). We will demonstrate this method on arbitrary features from the census data later on.

As before, we can also do another simple logic-based method to look for high probability but non-exact invariant-free inference. We can simply check for a condition like:

$$\sum_{i=1}^3 A_{Corr[i]} = A_{Parent} \pm 15$$

If we fulfill this condition, we can say that we are confident about the value of the attribute(s) (including the parent) to a probability of $\frac{3}{4}$. This is because it implies a result such as:

$$A_{Parent} = [30] \quad A_{Children} = [15, 15, 15]$$

Possible Solutions =

$$A_{Parent} = 33, A_{Children} = [11, 11, 11]$$

$$A_{Parent} = 34, A_{Children} = [12, 11, 11]$$

$$A_{Parent} = 34, A_{Children} = [11, 12, 11]$$

$$A_{Parent} = 34, A_{Children} = [11, 11, 12]$$

Again, this is the primary method we will use to look for invariant-free probabilistic inference vulnerabilities as opposed to the SAT solver method.

4 RESULTS AND STATISTICAL ANALYSIS

4.1 Exact Inference Results and Analysis

Please see Table 7 and Table 8 for results on invariant-based exact inference. Altogether, we found 285 sex(+) exact disclosures and 18 age exact solutions. Since our age attack reveals exactly 3 attributes and the sex(+) attack reveals two, we reveal exactly 624 attributes from the census disclosure using invariant-based exact inference. We found no examples of invariant-free exact inference.

To analyze the frequency of our attacks succeeding, we did some simple probabilistic calculations based on random rounding probability and validated these across the empirical results.

4.1.1 Analysis of Invariant-based Exact Inference. In order for this attack to succeed, it must be the case that all X attributes fall on the border of the same bound (upper/lower), but round to opposite sides. To estimate the probability of this happening, we multiply the probability of each attribute being at the bound ($1/5$, in mod 5), and then the probability of that attribute rounding to the opposite side ($1/5$ as well given the definition of random rounding). We must do this for all attributes in the candidate solution, and then multiply by two to consider both the upper and lower case.

In the sex+ case with only two rounded attributes, this works out to:

$$2 * (1/5)^4 = 0.32\% \text{ of instances, or 1-in-313.}$$

And in the age case with 3 attributes, it works out to:

$$2 * (1/5)^6 = 0.013\% \text{ of instances, or 1-in-7813.}$$

Remember that we are only considering regions where attributes are known to be above 10 since the behavior of sub-10 random rounding is not well defined for our attack. There are 61029 query-able census regions in total per our count, but only 61010 regions where we can apply our attack on sex+, and 59625 regions where we can apply our attack on age. Given our numbers, this predicts that we will find around 195 exact solutions to sex+ and 8 exact solutions to age from invariance-based exact inference. However, we discovered 285 exact solutions for sex+ and 18 exact

solutions for age respectively. This is within the relative range of our predictions, but the small discrepancy is worthy of future investigation.

4.1.2 Invariant-Free Exact Inference. We do a similar analysis to the above. However, we are now dealing with 5 attributes instead of 3. For the four sub-attributes, we require that they all take on a certain value at the bound (1/5, due to mod 5), and then round to the opposite bound (1/5 by random rounding). Next, the parent attribute must then round to the opposite bound (1/5). Finally, we account for both cases of the upper and lower-bound solutions, so we multiply by two. This gives a probability of:

$$2 * (1/5)^9 = 1.02 \times 10^{-4} \% \text{ of instances, or 1-in-976562.}$$

This seems inordinately small, but it is important to consider that this works for all groups of 4+1 attributes without the need for an invariant. We identify at least 20 such groups per census subdivision. After filtering out those groups which have some attributes possibly below 10, we still count 83898 total, which predicts around 0.0859 cases of exact inference in the full census disclosure from exact invariant-free inference. That makes the probability of this type of attack succeeding statistically unlikely but not altogether impossible. If we had an order of magnitude more municipalities or features, we might expect to find an instance of this attack being successful.

4.2 Probabilistic Inference

4.2.1 Invariant-Based Results. To demonstrate the SAT solver enumeration method, we can plot some histograms of numerical features from a sample census town (Colville Lake, in this case). To be interesting, we enumerate not only the three outer age categories but also the 3 subset age categories of age 0-14 (by enforcing their random rounding constraints as well as the constraint that they round to the age 0-14 solution). In this specific town, we can be 90% sure that there are either 27, 28, or 29 people aged 0-14.

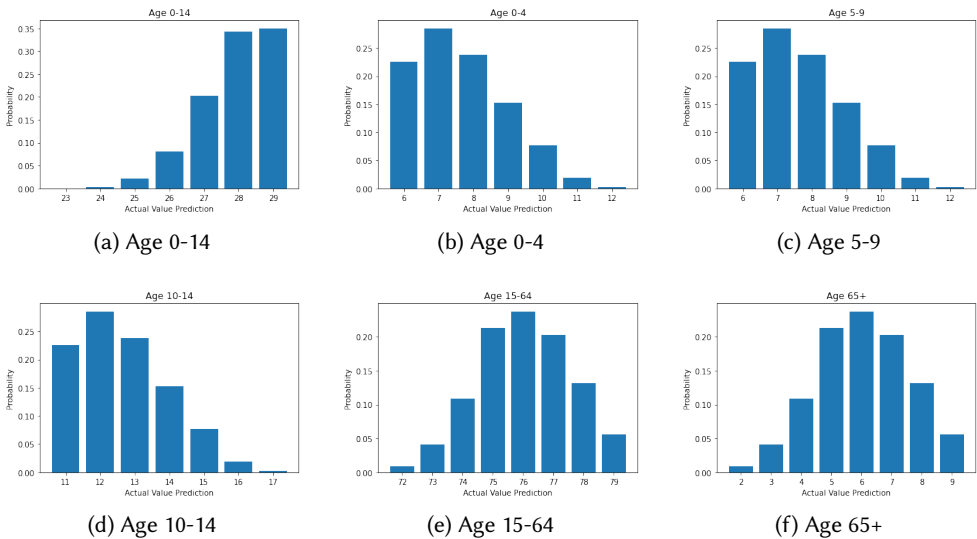


Fig. 2. Feature Probability Histograms

Next, we also check the census data for the condition we defined earlier. We will define a strong probabilistic result as being at least 66% confident in the value of any given attribute. By this definition, we examine the age attribute and find 83 examples of strong probabilistic inference in the 2021 Canadian census disclosure. See Table 9 for this result.

It’s unsurprising that this result is much more common than exact inference. In the age example, we are now allowed 3 different assignments of our three variables that still fulfill the condition, such as:

[11 11 12], [11 12 11], [11 11 12]

As well, random rounding is more likely to round to the other bound for the odd number out since it is closer to the opposite bound (2/5 odds rather than 1/5). Taking into account both directions of rounding (x2) and all 3 assignments (x3), we find the odds of this invariant-based strong probabilistic attack working on age as:

$$6 * (1/5)^5 * (2/5) = 7.68x10^{-2}\% \text{ of instances, or 1-in-1308.}$$

This is 6 times more likely than invariant-based exact inference attacks on age. (We will not examine sex(+) for this part, since it can only reach $\frac{1}{2}$ confidence with two sub-attributes for the non-exact case).

This predicts 48 instances of invariant-based strong probabilistic inference on age in the census dataset. Empirically, we find 83. Since each of these enumerates 3 sub-attributes, this is a total of 249 values inferred with strong probability.

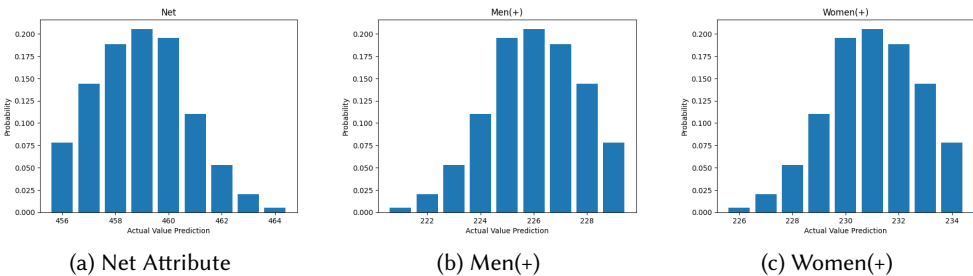
As a fun side note, the town of our university, Waterloo, is one of the regions exposed in this attack. We know that:

$$P[A_{Age0-14} = 17,644] = \frac{2}{3},$$

$$P[A_{Age15-64} = 85,084] = \frac{2}{3},$$

$$P[A_{Age65+} = 18,709] = \frac{2}{3}$$

4.2.2 Invariant-Free Probabilistic Inference. To demonstrate the SAT solver enumeration method, let us explore an arbitrary feature from the data; the number of married spouses or common-law partners in the town of Watson Lake. By using our exhaustive method, we are able to generate a feature probability density without the need for an invariant:



Again, this is an interesting result and proof of concept but proves to be computationally expensive (and non-revealing for the majority of cases). For this reason, we will mostly explore the census data with the logic condition-based method we defined earlier for invariant-free probabilistic inference.

Using the previous definition of a strong probabilistic result, we now explore mutually exclusive attribute categories with only 3 sub-attributes. We count 42 of these categories, as shown in Table 12. We find 216 cases of strong probabilistic inference (with $P=0.75$ probability in this case), which reveals 864 individual values with high probability (3 children and 1 parent each). We show these results in Table 10.

Again, it's unsurprising that this result is much more common than invariant-free exact inference. As explained before, we are allowed multiple assignments of variables that still fulfill the condition for the attack, and the overall random rounding probability is higher.

We first consider the case where the 3 attributes lie on the bound and round with $1/5$ probability, such that the summative parent attribute is not bounded and rounds to the opposite bound with $2/5$ probability. The second case is when one of the three attributes is one away from the bound as we have shown before. There are two directions of the bound and 3 possibilities per case. As well, the overall probability is the same as before because the subattribute rounds with $2/5$ probability whereas the parent attribute now rounds with $1/5$ probability.

We find the odds of this strong probabilistic attack working as:

$$(2 * (1/5)^6 * (2/5)) + (6 * (1/5)^6 * (2/5)) = 2.05 \times 10^{-2} \% \text{ of instances, or 1-in-4882.}$$

Since there are 918192 sets of attribute categories that are susceptible to this attack in the census disclosure this predicts 188 instances of invariant-free strong probabilistic inference on age in the census dataset. Empirically, we find 216, which is relatively close to the expected value.

5 SUGGESTED SOLUTION

There are a number of potential solutions to the issues highlighted in this paper.

One such solution is increasing the random rounding bound. For instance, increasing the rounding range would make extracting solutions, both probabilistically and exactly, numerically intractable and vanishingly unlikely. However, the utility of this solution is questionable. By using a range larger than 5, say 7, you now have non-intuitive bounds. 18 could round to 14 or 21, while 24 could round to 21 or 28. One of the major strengths of standard random rounding is interpretability since multiples of 5 are much easier to visualize and analyze than multiples of 7. As well, 15 could round all the way to 21, which is quite a long distance from the ground truth. By using a more intuitive bound like 10, the results once again become easily interpretable, but the utility is much less since any given value can be as much as 9 away in some cases.

Instead, we propose adding unbounded noise to the truth value. By doing this, it suddenly becomes impossible to say with certainty the range of values any specific attribute can take on, so the upper/lower bound exact inference attacks are no longer possible. It also makes SAT solver enumeration attacks intractable since it is generally no longer possible to enumerate all combinations of variables in an unbounded range.

For our solution, we specifically propose using the discrete Laplace distribution with the parameter $t=1.45$.

$$\mathbb{P}_{X \leftarrow \text{Lap}_Z(t)}[X = x] = \frac{e^{1/t} - 1}{e^{1/t} + 1} e^{-|x|/t} \text{ for all } x \in \mathbb{Z}$$

We choose the discrete Laplace partly because it is peaky and easily sampled. It also has appealing behavior under differential privacy [9], which allows for future extensions and experimentation into quantifiable privacy measures for the Canadian census.

We choose this specific t parameter value because the resulting distribution roughly mimics the random rounding PDF in terms of containing the majority of its probability density within 4 of the ground truth (remember that the greatest distance possible in mod-5 random rounding is 4) Specifically, this discrete Laplace with $t=1.45$ contains more than 95% of the expected sample utility within 4 of the true value, which roughly coincides with passing a hypothesis test of $p=0.05$ for being falling within the boundary of random rounding.

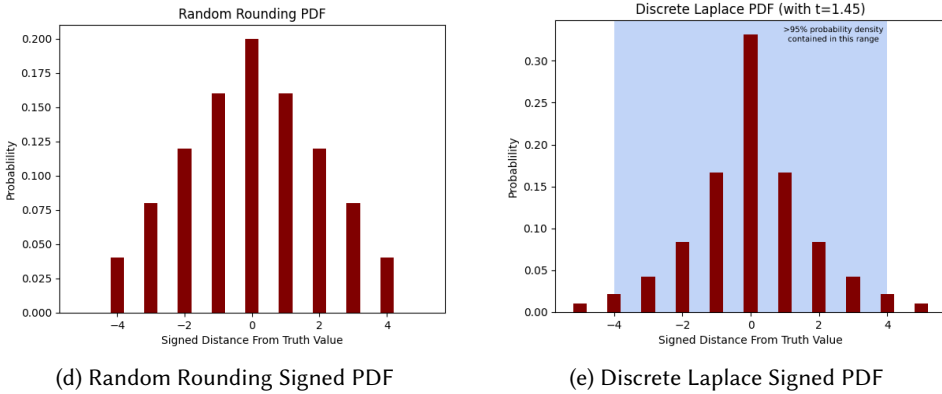


Fig. 3. Signed Distance PDFs

Although the privacy benefit is obvious, it is not immediately clear that the utility is comparable to random rounding. Interestingly, we can show that the utility of our approach is actually *superior* to regular random rounding in the average case. That is to say that, on average, our disclosed attribute values are closer to the true value than random rounding, while also having better privacy guarantees against exact inference. To show this, we will investigate the average (non-sub-10) unsigned distance from the true value by enumerating all possible cases as shown in Table 6. Notice that sample probability is equal for modular cases since the individual attributes of the exact cover are sampled independently and generally cover the mod 5 space with equal probability.

X mod 5	Sample Prob.	Distance Prob.	Sum Distance	Weighted Sum Distance
0	0.2	$1.0 * 0$	0	$0.2 * 0 = 0$
1	0.2	$(0.8 * -1) + (0.2 * 4)$	1.6	$0.2 * 1.6 = 0.32$
2	0.2	$(0.6 * -2) + (0.4 * 3)$	2.4	$0.2 * 2.4 = 0.48$
3	0.2	$(0.4 * -3) + (0.6 * 2)$	2.4	$0.2 * 2.4 = 0.48$
4	0.2	$(0.2 * -4) + (0.8 * 1)$	1.6	$0.2 * 1.6 = 0.32$
				Sum Weighted Distance = 1.6

Table 6. Determining Sum Weighted Distance Utility for Random Rounding

We cannot get an exact closed-form distance for the proposed method because the range of the discrete Laplace is unbounded. However, we can estimate a very precise analytical distance by

summing the discrete Laplace over a generous range (-10 to 10, for example). The sampled noise value is essentially guaranteed to be within this range (> 99.9% of samples are within this range). What we find is that the average weighted distance is only 1.33, which is approximately 20% closer than the 1.6 average distance given by random rounding. However, unlike in random rounding, it is no longer possible to enumerate edge cases such that we can say with certainty the actual truth values.

Our proposed method may need additional thought when it comes to the zero case since disclosing a value as below zero is non-intuitive and clearly shows an impossible solution. However, we naively suggest either 1) leaving it as is, since exact inference is still impossible even in this case, or 2) clamping the disclosure at 0, such that all published attribute values are non-negative. Further investigation is warranted here if this was explored as a real possibility.

6 CONCLUSION AND FUTURE WORK

A natural question to ask is if these leaked attributes pose any kind of security or privacy risk. This is an open question. It is the authors' opinion that, since these vulnerabilities allow for the reconstruction of non-invariant attributes, they are worthy of patching. It's also a possibility that the exact values obtained from these attacks could be used for more sophisticated attacks. At the present moment, these findings present a vulnerability in the methodology but not an imminent personal risk.

We also believe that we have not nearly exhausted the possibilities for attacking the random rounding mechanism. There are likely more creative ways to perform both exact and probabilistic inference which have not yet been thought out. As well, we chose to ignore sub-10 random rounding because it behaves slightly differently than typical mod-5 random rounding. In the future, we would like to remove this constraint as it would open up a vast number of possibilities for further exploitation.

Next, we plan to investigate extending our attacks into revealing more discrete information about the underlying dataset. For instance, using SAT solvers to identify or reconstruct the characteristics of certain individuals in the dataset (like the exact ages of people in a region). This is a much more devastating attack and has direct and dire consequences if this type of information can be extracted from the publicly-available census disclosure. We believe this type of attack might be possible with more sophisticated methods, such as by leveraging the median and mean attributes that are published alongside the counting attributes.

Overall, we have demonstrated a potential vulnerability in the 2021 Canadian census disclosure, and encourage further research in this area to ensure confidentiality and trust in the security of all future census disclosures.

ACKNOWLEDGMENTS

We thank Prof. Gautam Kamath for editing and help with privacy concepts. Thanks to Candace Trusty from Statistics Canada for clarifying the Canadian census's practices.

REFERENCES

- [1] John M. Abowd. 2018. *Staring-Down the Database Reconstruction Theorem*. Retrieved July 25, 2023 from <https://www.census.gov/content/dam/Census/newsroom/press-kits/2018/jsm/jsm-presentation-database-reconstruction.pdf>
- [2] John M Abowd, Robert Ashmead, Ryan Cumings-Menon, Simson Garfinkel, Micah Heineck, Christine Heiss, Robert Johns, Daniel Kifer, Philip Leclerc, Ashwin Machanavajjhala, et al. 2022. The 2020 Census Disclosure Avoidance System TopDown Algorithm. *arXiv preprint arXiv:2204.08986* (2022).
- [3] Canadian Census Bureau. 2022. *Census Profile, 2021 Census of Population*. Retrieved June 6, 2023 from <https://www12.statcan.gc.ca/census-recensement/2021/dp-pd/prof/details/download-telecharger.cfm?Lang=E>

- [4] US Census Bureau. 2020. *Invariants Set for 2020 Census Data Products*. Retrieved June 6, 2023 from <https://www.census.gov/programs-surveys/decennial-census/decade/2020/planning-management/process/disclosure-avoidance/2020-das-updates/2020-11-25.html>
- [5] United States Census Bureau. 2023. *US Census Privacy Policy*. Retrieved July 25, 2023 from <https://www.census.gov/about/policies/privacy/privacy-policy.html>
- [6] Canada. 1985. Privacy Act. <https://laws-lois.justice.gc.ca/PDF/P-21.pdf> Last amended 1 October 2022. Accessed 21 December 2022.
- [7] Canada. 1985. Statistics Act. <https://laws-lois.justice.gc.ca/PDF/S-19.pdf> Last amended 12 December 2017. Accessed 21 December 2022.
- [8] Statistics Canada. 2022. *Guide to the Census of Population, 2021 Chapter 10 – Dissemination*. Retrieved December 19, 2022 from <https://www12.statcan.gc.ca/census-recensement/2021/ref/98-304/2021001/chap10-eng.cfm>
- [9] Clément L. Canonne, Gautam Kamath, and Thomas Steinke. 2020. The Discrete Gaussian for Differential Privacy. *CoRR* abs/2004.00010 (2020). arXiv:2004.00010 <https://arxiv.org/abs/2004.00010>
- [10] Z3 development team. 2022. *Z3 Solver*. <https://github.com/Z3Prover/z3>
- [11] Simson Garfinkel, John M. Abowd, and Christian Martindale. 2019. Understanding Database Reconstruction Attacks on Public Data. *Commun. ACM* 62, 3 (feb 2019), 46–53. <https://doi.org/10.1145/3287287>
- [12] Simson L. Garfinkel, John M. Abowd, and Chris Martindale. 2018. Understanding Database Reconstruction Attacks on Public Data. *Queue* 16 (2018), 28 – 53.
- [13] Erica L. Groshen and Daniel Goroff. 2022. *Disclosure Avoidance and the 2020 Census: What Do Researchers Need to Know?* Retrieved July 25, 2023 from <https://hdsr.mitpress.mit.edu/pub/tgi3gol2/release/2>

A APPENDIX

A.1 Non-Exact Inference

Table 7. All Discovered Age Exact Inference Disclosures

Region / Municipality	Pop (Invariant)	A0-14 (Pub)	A15-64 (Pub)	A65+ (Pub)	A0-14 (Real)	A15-64 (Real)	A65+ (Real)
35180801	2192	390	1275	515	394	1279	519
35190780	3882	745	2760	365	749	2764	369
35290263	863	155	575	145	151	571	141
35310157	552	75	320	145	79	324	149
Chippewas of Rama First ...	998	170	700	140	166	696	136
35560318	412	30	195	175	34	199	179
12090879	643	90	475	90	86	471	86
12100084	457	55	300	90	59	304	94
12110069	382	50	245	75	54	249	79
47060411	498	115	345	50	111	341	46
48062615	2268	620	1440	220	616	1436	216
48070161	498	80	315	115	76	311	111
48112456	157	20	85	40	24	89	44
48110853	257	25	195	25	29	199	29
48150153	702	90	425	175	94	429	179
24230246	1272	125	820	315	129	824	319
24530113	573	95	365	125	91	361	121
24710197	548	100	330	130	96	326	126

Table 8. All Discovered Sex(+) Exact Inference Disclosures

Region / Municipality	Pop (Invariant)	Published Male+	Published Fem+	Real Male+	Real Fem+
35010267	412	200	220	196	216
35060033	398	205	185	209	189
35060864	638	330	300	334	304
35061314	618	310	300	314	304
35070195	597	305	300	301	296
35070324	488	235	245	239	249
35090110	907	410	505	406	501
35100194	502	220	290	216	286
35100303	598	285	305	289	309
35110113	443	220	215	224	219
35110072	648	325	315	329	319
35150281	657	335	330	331	326
35160108	593	285	300	289	304
35160164	952	470	490	466	486
35160190	538	220	310	224	314
35190189	568	280	280	284	284
35190219	272	140	140	136	136
35190393	738	370	360	374	364
35190700	9737	4795	4950	4791	4946
Newmarket, Town (T)	87942	42560	45390	42556	45386
35190916	732	385	355	381	351
35200201	567	270	305	266	301
35200244	387	205	190	201	186
35200582	763	365	390	369	394
35200932	258	125	125	129	129
35200960	393	190	195	194	199
35201106	493	220	265	224	269
35201567	668	315	345	319	349
35201829	472	240	240	236	236
35202406	897	405	500	401	496

35203753	347	175	180	171	176
35204473	423	205	210	209	214
35204671	422	175	255	171	251
35204769	1518	735	775	739	779
35210244	473	230	235	234	239
35210398	388	195	185	199	189
35210501	808	405	395	409	399
35210758	497	245	260	241	256
35210036	452	230	230	226	226
35212041	513	260	245	264	249
35211612	712	365	355	361	351
35220157	522	265	265	261	261
35230057	657	320	345	316	341
35230430	453	230	215	234	219
35240233	622	310	320	306	316
35240701	563	275	280	279	284
35250399	788	465	315	469	319
35260177	587	275	320	271	316
35260439	483	225	250	229	254
35260583	742	340	410	336	406
35260300	592	280	320	276	316
35260314	407	210	205	206	201
35290271	407	220	195	216	191
35290304	1513	735	770	739	774
35300466	613	315	290	319	294
35300035	1348	685	655	689	659
35310164	548	285	255	289	259
35310226	1308	615	685	619	689
35320224	433	215	210	219	214
35360354	398	210	180	214	184
35370813	822	395	435	391	431
35370737	578	285	285	289	289
35370678	1058	525	525	529	529
35370650	858	445	405	449	409
35380081	403	200	195	204	199
35380119	398	185	205	189	209
35390394	1723	820	895	824	899
35420257	542	235	315	231	311
35430926	588	285	295	289	299
35431395	5337	2670	2675	2666	2671
35430706	677	340	345	336	341
35430613	567	295	280	291	276
35430564	567	270	305	266	301
Midland, Town (T)	17817	8430	9395	8426	9391
35440239	853	735	110	739	114
35440221	672	340	340	336	336
35470213	518	245	265	249	269
35470164	692	355	345	351	341
West Nipissing / Nipissing Ouest, ...	14583	7140	7435	7144	7439
35490154	518	255	255	259	259
35520138	392	215	185	211	181
35530083	712	420	300	416	296
35530135	612	300	320	296	316
35530170	773	365	400	369	404
35530369	588	300	280	304	284
35560295	398	200	190	204	194
35560299	362	175	195	171	191
35580485	618	320	290	324	294
Thunder Bay, City (CY)	108843	53505	55330	53509	55334
35580078	548	275	265	279	269

35580152	603	340	255	344	259
35580466	603	285	310	289	314
35600396	533	260	265	264	269
60010210	437	210	235	206	231
61010047	162	75	95	71	91
62040061	3087	1540	1555	1536	1551
62050021	1277	655	630	651	626
10010511	417	195	230	191	226
10010526	323	155	160	159	164
10010572	1517	745	780	741	776
10010573	1302	600	710	596	706
Northport, Rural municipality (RM)	157	80	85	76	81
11030240	157	80	85	76	81
12020057	447	200	255	196	251
12060177	137	70	75	66	71
East Hants, Municipal district (MD)	22892	11425	11475	11421	11471
12110075	377	190	195	186	191
12170453	598	275	315	279	319
12180030	293	130	155	134	159
13010085	548	275	265	279	269
13020094	467	215	260	211	256
13030054	417	215	210	211	206
13050115	602	320	290	316	286
13050121	373	185	180	189	184
13100242	497	265	240	261	236
13140114	482	245	245	241	241
13150304	472	235	245	231	241
13150320	317	150	175	146	171
13150168	502	245	265	241	261
Central Kootenay J, Regional district ...	3517	1820	1705	1816	1701
59050111	528	260	260	264	264
59070283	488	255	225	259	229
59070318	562	275	295	271	291
59090123	1223	595	620	599	624
59090724	412	210	210	206	206
59090894	567	270	305	266	301
59090897	602	270	340	266	336
59090920	888	445	435	449	439
Langley, District municipality (DM)	132603	64830	67765	64834	67769
59152281	498	240	250	244	254
59152322	437	240	205	236	201
59152644	702	340	370	336	366
59154203	1137	565	580	561	576
59154279	598	225	365	229	369
59152740	553	280	265	284	269
59151084	453	215	230	219	234
59150567	517	205	320	201	316
59150688	528	235	285	239	289
59150827	513	250	255	254	259
59151198	522	295	235	291	231
59150274	567	305	270	301	266
59153496	587	275	320	271	316
59153693	443	220	215	224	219
59153694	473	245	220	249	224
59153549	117	60	65	56	61
59154007	883	430	445	434	449
59150228	1218	605	605	609	609
59150045	622	270	360	266	356
59170240	697	345	360	341	356
59170265	448	220	220	224	224

59170414	1297	625	680	621	676
59210400	657	310	355	306	351
59210287	512	265	255	261	251
59210443	522	270	260	266	256
59240218	517	285	240	281	236
59260296	852	430	430	426	426
59270057	478	245	225	249	229
59350217	1553	660	885	664	889
59350381	1422	715	715	711	711
59350452	1142	565	585	561	581
59370160	457	230	235	226	231
59370266	73	35	30	39	34
59370298	432	225	215	221	211
59390219	497	250	255	246	251
59390227	587	310	285	306	281
59410480	348	170	170	174	174
59430070	442	230	220	226	216
Bulkley-Nechako A, Regional district ...	5587	2905	2690	2901	2686
59530044	463	240	215	244	219
59530085	242	120	130	116	126
Pouce Coupe, Village (VL)	762	370	400	366	396
59550145	422	210	220	206	216
46020088	757	390	375	386	371
46020126	602	290	320	286	316
46090089	453	230	215	234	219
46110538	677	350	335	346	331
46110658	468	260	200	264	204
46110672	552	300	260	296	256
Grahamdale, Rural municipality (RM)	1278	665	605	669	609
Invermay, Village (VL)	272	140	140	136	136
47090230	272	140	140	136	136
Aquadeo, Resort village (RV)	203	85	110	89	114
47170266	203	85	110	89	114
48020136	497	255	250	251	246
48020434	3377	1675	1710	1671	1706
48062058	667	350	325	346	321
48060616	538	265	265	269	269
48061285	332	175	165	171	161
48061490	473	240	225	244	229
48061606	382	195	195	191	191
48061617	757	385	380	381	376
48062234	933	450	475	454	479
48062307	917	375	550	371	546
48062334	443	290	145	294	149
48070205	433	215	210	219	214
48080313	438	220	210	224	214
48080128	438	220	210	224	214
48080154	267	150	125	146	121
48080457	353	155	190	159	194
Bentley, Town (T)	1042	540	510	536	506
48080398	453	220	225	224	229
48080439	498	270	220	274	224
48090063	578	285	285	289	289
48100202	548	285	255	289	259
48100353	387	185	210	181	206
48112068	517	270	255	266	251
48111772	793	390	395	394	399
48112049	803	410	385	414	389
48110364	477	260	225	256	221
48110871	563	280	275	284	279

48111026	433	215	210	219	214
48111452	507	255	260	251	256
48111589	538	245	285	249	289
48112528	682	355	335	351	331
48112538	1152	595	565	591	561
48120200	897	440	465	436	461
48120140	617	300	325	296	321
Sandy Beach, Summer village (SV)	278	135	135	139	139
48130240	278	135	135	139	139
48190313	277	155	130	151	126
24040037	713	345	360	349	364
24170034	622	310	320	306	316
24210057	357	180	185	176	181
24230067	1018	470	540	474	544
24230099	327	170	165	166	161
24230195	377	195	190	191	186
24230435	403	195	200	199	204
24230668	548	280	260	284	264
24230801	417	205	220	201	216
24230910	1697	840	865	836	861
24230923	672	345	335	341	331
24231043	448	215	225	219	229
24230506	797	390	415	386	411
24250277	1537	770	775	766	771
24310080	757	365	400	361	396
24360142	488	265	215	269	219
24360144	647	315	340	311	336
24370290	403	210	185	214	189
24390098	362	205	165	201	161
24390162	368	180	180	184	184
24430254	447	215	240	211	236
Saint-Armand, Municipalité (MÉ)	1228	620	600	624	604
24470107	467	250	225	246	221
Maskinongé, Municipalité (MÉ)	2323	1210	1105	1214	1109
24560287	1803	865	930	869	934
24570148	477	260	225	256	221
24570248	932	475	465	471	461
24580308	818	395	415	399	419
24580747	1563	765	790	769	794
24590183	643	330	305	334	309
24610103	738	345	385	349	389
24620185	437	225	220	221	216
24640141	1897	930	975	926	971
24640304	1018	520	490	524	494
24650498	697	340	365	336	361
24660006	417	215	210	211	206
24660149	1132	515	625	511	621
24661257	818	430	380	434	384
24661542	367	195	180	191	176
24661609	1057	505	560	501	556
24661635	522	265	265	261	261
24661686	702	350	360	346	356
24662011	757	350	415	346	411
24662341	782	395	395	391	391
24662543	272	140	140	136	136
24663218	668	310	350	314	354
24663353	658	320	330	324	334
24663358	1012	495	525	491	521
24670136	408	210	190	214	194
24690066	718	315	395	319	399

24700090	472	245	235	241	231
24700154	1043	525	510	529	514
24710150	412	215	205	211	201
24730089	467	240	235	236	231
24780158	507	260	255	256	251
24780123	492	220	280	216	276
24790122	673	330	335	334	339
24810446	653	345	300	349	304
24850062	478	230	240	234	244
24930131	647	330	325	326	321
24940041	522	225	305	221	301
24940457	568	305	255	309	259
24940460	1438	690	740	694	744
Saint-Ambroise, Municipalité (MÉ)	3883	2030	1845	2034	1849
Chapais, Ville (V)	1468	770	690	774	694

Table 9. All Discovered Age Strong Probabilistic Inference Disclosures

Region / Municipality	Pop (Invariant)	Published			Strong Probs (P >= 0.66)		
		A0-14	A15-64	A65+	A0-14	A15-64	A65+
35010302	629	115	340	185	111	336	181
35060647	381	75	245	50	79	249	54
35070179	511	65	325	110	69	329	114
35120390	416	80	250	75	84	254	79
35150085	559	75	350	145	71	346	141
35160132	386	35	220	120	39	224	124
35180190	496	65	385	35	69	389	39
35191022	1259	130	820	320	126	816	316
35190080	471	90	270	100	94	274	104
35200368	339	45	240	65	41	236	61
35201694	664	85	460	130	81	456	126
35210271	631	55	420	145	59	424	149
35210791	244	40	160	55	36	156	51
35210049	564	85	410	80	81	406	76
35210071	316	65	220	20	69	224	24
35241033	789	95	355	350	91	351	346
35250081	989	185	675	140	181	671	136
35250099	784	90	365	340	86	361	336
35300462	1059	155	640	275	151	636	271
Waterloo, City (CY)	121436	17640	85080	18705	17644	85084	18709
35370767	389	65	265	70	61	261	66
35370131	586	195	320	60	199	324	64
35370200	466	90	305	60	94	309	64
35370201	521	120	340	50	124	344	54
35390144	1501	280	810	400	284	814	404
35390336	614	65	380	180	61	376	176
35390659	791	110	555	115	114	559	119
35420185	409	70	265	85	66	261	81
35430745	789	135	515	150	131	511	146
35560293	456	50	270	125	54	274	129
35590107	484	70	305	120	66	301	116
12090597	894	160	620	125	156	616	121
12090611	416	55	265	85	59	269	89
Cumberland, Subd. B, Subdivision of county munic...	6786	810	4160	1805	814	4164	1809
12140075	421	70	235	105	74	239	109
12140077	1001	170	635	185	174	639	189
12170385	571	60	315	185	64	319	189
13080083	484	100	330	65	96	326	61

13150279	444	60	285	110	56	281	106
59090923	506	75	330	90	79	334	94
59090928	1036	120	780	125	124	784	129
59090510	664	150	425	100	146	421	96
59150340	534	65	390	90	61	386	86
59153296	686	70	475	130	74	479	134
59150264	319	50	225	55	46	221	51
59190160	469	80	315	85	76	311	81
59210454	1701	225	1080	385	229	1084	389
59310176	1764	145	1470	160	141	1466	156
59370141	524	35	195	305	31	191	301
Burns Lake, Village (VL)	1659	310	1020	340	306	1016	336
Fort Nelson 2, Indian reserve (IRI)	419	85	275	70	81	271	66
59590038	419	85	275	70	81	271	66
46030086	716	155	425	125	159	429	129
46040043	411	90	225	85	94	229	89
46100020	556	90	345	110	94	349	114
46100024	664	130	450	95	126	446	91
46110411	436	35	365	25	39	369	29
46190124	464	190	250	35	186	246	31
48010230	1146	240	745	150	244	749	154
48130288	1006	210	720	65	214	724	69
Manning, Town (T)	1126	200	670	245	204	674	249
24130061	441	40	255	135	44	259	139
24230383	686	115	415	145	119	419	149
24230920	736	125	430	170	129	434	174
24231011	1566	520	955	80	524	959	84
24320074	446	55	250	130	59	254	134
24370132	239	20	170	60	16	166	56
24430092	276	15	165	85	19	169	89
24580050	531	85	335	100	89	339	104
Calixa-Lavallée, Municipalité (MÉ)	509	75	335	110	71	331	106
24590191	509	75	335	110	71	331	106
24662542	879	165	580	145	161	576	141
24670258	536	120	375	30	124	379	34
24720109	756	185	490	70	189	494	74
24730046	486	45	295	135	49	299	139
24750050	384	50	240	105	46	236	101
24770069	986	110	650	215	114	654	219
24810094	756	175	520	50	179	524	54
24850067	541	120	325	85	124	329	89
24890110	466	60	300	95	64	304	99
24910055	594	120	355	130	116	351	126
24920078	321	50	185	75	54	189	79
24940241	561	60	275	215	64	279	219

Table 10. All Discovered Invariant-Free Strong Probabilistic Inferences

Region / Municipality	Attributes	Sex(+)	Published				Strong Probs (P = 0.75 >0.66)			
			A1	A2	A3	A4	A1	A2	A3	A4
35020178	[335, 336, 338, 339]	F(+)	245	25	95	110	241	29	99	114
35060073	[335, 336, 338, 339]	F(+)	160	35	85	55	164	31	81	51
35060588	[1452, 1453, 1454, 1455]	All	405	220	145	25	401	224	149	29
35061091	[350, 351, 353, 354]	F(+)	225	45	135	60	229	41	131	56
35061298	[1452, 1453, 1454, 1455]	All	310	145	165	15	314	141	161	11
35061316	[350, 351, 353, 354]	M(+)	160	30	115	30	164	26	111	26
35061848	[350, 351, 353, 354]	F(+)	360	85	250	40	364	81	246	36
35110128	[335, 336, 338, 339]	M(+)	535	105	275	170	539	101	271	166

35160208	[340, 341, 343, 344]	All	160	50	80	15	156	54	84	19
35180912	[1665, 1666, 1667, 1668]	All	500	220	175	90	496	224	179	94
35180217	[350, 351, 353, 354]	F(+)	280	50	185	60	284	46	181	56
35180667	[335, 336, 338, 339]	All	510	90	315	90	506	94	319	94
35180772	[350, 351, 353, 354]	All	515	110	300	90	511	114	304	94
35190802	[335, 336, 338, 339]	M(+)	160	20	90	35	156	24	94	39
35191289	[350, 351, 353, 354]	All	11190	3195	7015	965	11186	3199	7019	969
35190637	[335, 336, 338, 339]	M(+)	185	30	130	40	189	26	126	36
35191205	[335, 336, 338, 339]	M(+)	185	45	135	20	189	41	131	16
35190998	[335, 336, 338, 339]	All	395	70	265	75	399	66	261	71
35190884	[350, 351, 353, 354]	M(+)	175	50	120	20	179	46	116	16
35200245	[1452, 1453, 1454, 1455]	All	240	95	105	25	236	99	109	29
35200464	[335, 336, 338, 339]	All	670	125	410	120	666	129	414	124
35201034	[1665, 1666, 1667, 1668]	F(+)	90	30	20	25	86	34	24	29
35201114	[335, 336, 338, 339]	All	500	75	380	60	504	71	376	56
35201210	[350, 351, 353, 354]	M(+)	130	20	100	25	134	16	96	21
35201723	[1452, 1453, 1454, 1455]	All	130	45	65	35	134	41	61	31
35203279	[335, 336, 338, 339]	F(+)	310	40	185	70	306	44	189	74
35203471	[340, 341, 343, 344]	All	60	15	35	25	64	11	31	21
35204825	[335, 336, 338, 339]	F(+)	510	40	450	35	514	36	446	31
35210842	[350, 351, 353, 354]	All	975	145	680	165	979	141	676	161
35210912	[335, 336, 338, 339]	All	660	90	455	130	664	86	451	126
35211070	[335, 336, 338, 339]	All	450	100	280	85	454	96	276	81
35211071	[335, 336, 338, 339]	All	450	100	280	85	454	96	276	81
35211923	[335, 336, 338, 339]	F(+)	425	95	300	45	429	91	296	41
35212276	[335, 336, 338, 339]	F(+)	200	45	140	30	204	41	136	26
35210061	[335, 336, 338, 339]	F(+)	305	60	195	35	301	64	199	39
35210166	[335, 336, 338, 339]	F(+)	195	40	130	40	199	36	126	36
35211122	[335, 336, 338, 339]	All	845	170	555	105	841	174	559	109
35211498	[335, 336, 338, 339]	F(+)	425	55	275	80	421	59	279	84
35211806	[335, 336, 338, 339]	F(+)	190	25	125	25	186	29	129	29
35211899	[335, 336, 338, 339]	All	515	135	345	20	511	139	349	24
35220182	[335, 336, 338, 339]	M(+)	330	75	200	40	326	79	204	44
35240663	[335, 336, 338, 339]	M(+)	245	45	150	35	241	49	154	39
35240664	[350, 351, 353, 354]	All	615	135	340	155	619	131	336	151
35240055	[335, 336, 338, 339]	All	360	65	225	55	356	69	229	59
35250051	[335, 336, 338, 339]	F(+)	580	90	330	145	576	94	334	149
35250073	[90, 91, 92, 93]	F(+)	180	105	20	70	184	101	16	66
35250334	[350, 351, 353, 354]	M(+)	270	40	190	55	274	36	186	51
35250527	[90, 91, 92, 93]	All	375	220	35	135	379	216	31	131
35251029	[335, 336, 338, 339]	All	7625	2305	4795	540	7629	2301	4791	536
35260038	[335, 336, 338, 339]	M(+)	225	25	150	35	221	29	154	39
35260563	[335, 336, 338, 339]	M(+)	180	35	100	30	176	39	104	34
35280337	[335, 336, 338, 339]	F(+)	205	50	120	20	201	54	124	24
New Credit (Part) 40A, Indian...	[335, 336, 338, 339]	F(+)	205	50	120	20	201	54	124	24
35280165	[335, 336, 338, 339]	F(+)	205	50	120	20	201	54	124	24
Six Nations (Part) 40, Indian...	[335, 336, 338, 339]	F(+)	205	50	120	20	201	54	124	24
35280163	[335, 336, 338, 339]	F(+)	205	50	120	20	201	54	124	24
35300484	[340, 341, 343, 344]	All	45	15	30	15	49	11	26	11
35300649	[1452, 1453, 1454, 1455]	All	610	195	380	20	606	199	384	24
35300191	[350, 351, 353, 354]	F(+)	140	25	105	25	144	21	101	21
35300981	[90, 91, 92, 93]	All	530	280	50	185	526	284	54	189
35300013	[335, 336, 338, 339]	F(+)	225	50	130	60	229	46	126	56
35340198	[1665, 1666, 1667, 1668]	F(+)	295	35	40	205	291	39	44	209
35340058	[90, 91, 92, 93]	F(+)	305	175	20	95	301	179	24	99
Southwold, Township (TP)	[1452, 1453, 1454, 1455]	All	1710	650	1000	75	1714	646	996	71
35360251	[90, 91, 92, 93]	All	570	280	95	210	574	276	91	206
35360463	[350, 351, 353, 354]	M(+)	170	15	100	40	166	19	104	44
35360464	[350, 351, 353, 354]	M(+)	170	15	100	40	166	19	104	44
35360465	[350, 351, 353, 354]	M(+)	170	15	100	40	166	19	104	44

35370804	[90, 91, 92, 93]	All	465	220	25	205	461	224	29	209
35370096	[335, 336, 338, 339]	F(+)	245	50	130	80	249	46	126	76
35370667	[1452, 1453, 1454, 1455]	All	400	160	185	40	396	164	189	44
35370608	[1665, 1666, 1667, 1668]	All	415	80	70	250	411	84	74	254
35380081	[335, 336, 338, 339]	M(+)	200	40	145	30	204	36	141	26
35380174	[335, 336, 338, 339]	F(+)	335	50	175	125	339	46	171	121
35380286	[350, 351, 353, 354]	All	720	190	380	135	716	194	384	139
35390110	[350, 351, 353, 354]	F(+)	240	35	155	35	236	39	159	39
35400139	[350, 351, 353, 354]	M(+)	215	45	120	35	211	49	124	39
Kincardine, Municipality (MU)	[335, 336, 338, 339]	All	12065	2505	6805	2740	12061	2509	6809	2744
35420296	[340, 341, 343, 344]	F(+)	60	20	40	15	64	16	36	11
35430801	[335, 336, 338, 339]	F(+)	300	55	190	40	296	59	194	44
35430743	[1452, 1453, 1454, 1455]	All	265	80	155	15	261	84	159	19
35530087	[335, 336, 338, 339]	F(+)	280	30	190	45	276	34	194	49
35530300	[350, 351, 353, 354]	M(+)	385	80	225	65	381	84	229	69
35540125	[350, 351, 353, 354]	All	465	90	275	115	469	86	271	111
Huron Shores, Municipality (MU)	[1452, 1453, 1454, 1455]	All	830	420	395	30	834	416	391	26
35570339	[335, 336, 338, 339]	M(+)	205	25	105	60	201	29	109	64
35570397	[350, 351, 353, 354]	M(+)	340	75	190	90	344	71	186	86
Norman's Cove-Long Cove ...	[350, 351, 353, 354]	F(+)	320	30	170	105	316	34	174	109
10010717	[350, 351, 353, 354]	F(+)	320	30	170	105	316	34	174	109
10010497	[335, 336, 338, 339]	M(+)	220	20	120	65	216	24	124	69
10010499	[335, 336, 338, 339]	M(+)	220	20	120	65	216	24	124	69
10010544	[335, 336, 338, 339]	All	545	120	305	105	541	124	309	109
10010663	[350, 351, 353, 354]	M(+)	160	20	85	40	156	24	89	44
10010664	[350, 351, 353, 354]	M(+)	160	20	85	40	156	24	89	44
St. Alban's, Town (T)	[1665, 1666, 1667, 1668]	F(+)	625	15	15	610	629	11	11	606
Middle Arm, Town (T)	[340, 341, 343, 344]	M(+)	50	15	20	30	54	11	16	26
10080220	[340, 341, 343, 344]	M(+)	50	15	20	30	54	11	16	26
Tilt Cove, Town (T)	[340, 341, 343, 344]	M(+)	50	15	20	30	54	11	16	26
10080223	[340, 341, 343, 344]	M(+)	50	15	20	30	54	11	16	26
Division No. 11, Census div...	[90, 91, 92, 93]	All	1970	910	180	895	1974	906	176	891
Division No. 11, Subd. C, Subdiv...	[90, 91, 92, 93]	All	1970	910	180	895	1974	906	176	891
10110005	[90, 91, 92, 93]	All	1970	910	180	895	1974	906	176	891
12020043	[350, 351, 353, 354]	M(+)	405	70	220	100	401	74	224	104
12090121	[335, 336, 338, 339]	F(+)	355	75	235	60	359	71	231	56
12090486	[335, 336, 338, 339]	M(+)	220	20	155	30	216	24	159	34
12090780	[335, 336, 338, 339]	M(+)	300	55	180	50	296	59	184	54
12090788	[1452, 1453, 1454, 1455]	All	195	95	70	15	191	99	74	19
12100080	[350, 351, 353, 354]	F(+)	270	45	130	80	266	49	134	84
12140072	[335, 336, 338, 339]	M(+)	255	55	150	65	259	51	146	61
13010053	[350, 351, 353, 354]	All	500	60	385	40	496	64	389	44
Pennfield, Parish (P)	[335, 336, 338, 339]	All	2195	390	1335	485	2199	386	1331	481
13020094	[335, 336, 338, 339]	M(+)	215	30	130	40	211	34	134	44
13050205	[335, 336, 338, 339]	F(+)	260	30	155	60	256	34	159	64
13070116	[1452, 1453, 1454, 1455]	All	360	205	115	25	356	209	119	29
York, County (CT)	[2054, 2055, 2056, 2057]	M(+)	3490	15	210	3280	3494	11	206	3276
13100299	[340, 341, 343, 344]	F(+)	35	15	20	15	39	11	16	11
13100189	[335, 336, 338, 339]	M(+)	220	60	140	35	224	56	136	31
59030128	[335, 336, 338, 339]	F(+)	255	30	160	80	259	26	156	76
59090375	[335, 336, 338, 339]	All	545	135	330	65	541	139	334	69
59153784	[335, 336, 338, 339]	M(+)	325	55	210	75	329	51	206	71
59152015	[335, 336, 338, 339]	All	585	130	340	100	581	134	344	104
59153504	[335, 336, 338, 339]	M(+)	390	85	235	55	386	89	239	59
59154222	[335, 336, 338, 339]	M(+)	200	45	135	35	204	41	131	31
59152671	[335, 336, 338, 339]	M(+)	315	85	190	55	319	81	186	51
59153451	[350, 351, 353, 354]	F(+)	400	60	255	100	404	56	251	96
59152860	[1452, 1453, 1454, 1455]	All	585	400	155	15	581	404	159	19
59153038	[1452, 1453, 1454, 1455]	All	580	255	290	50	584	251	286	46
59150136	[350, 351, 353, 354]	All	510	115	340	70	514	111	336	66

59150158	[335, 336, 338, 339]	All	540	95	340	90	536	99	344	94
59150159	[335, 336, 338, 339]	All	650	155	415	95	654	151	411	91
59152462	[1452, 1453, 1454, 1455]	All	265	150	110	20	269	146	106	16
59190246	[1665, 1666, 1667, 1668]	M(+)	195	35	20	125	191	39	24	129
59260370	[90, 91, 92, 93]	F(+)	235	145	20	55	231	149	24	59
59350253	[340, 341, 343, 344]	All	145	25	65	70	149	21	61	66
59410388	[1452, 1453, 1454, 1455]	All	380	180	190	25	384	176	186	21
59490129	[350, 351, 353, 354]	All	425	125	255	30	421	129	259	34
Peace River, Regional district (RD)	[96, 97, 98, 99]	M(+)	6615	4160	655	1815	6619	4156	651	1811
59550194	[1665, 1666, 1667, 1668]	F(+)	250	15	40	210	254	11	36	206
46030148	[335, 336, 338, 339]	All	1725	550	1000	160	1721	554	1004	164
46110177	[1452, 1453, 1454, 1455]	All	275	180	95	15	279	176	91	11
46110219	[90, 91, 92, 93]	All	635	375	25	250	639	371	21	246
46110824	[350, 351, 353, 354]	M(+)	250	55	150	30	246	59	154	34
46210100	[1452, 1453, 1454, 1455]	All	210	125	55	15	206	129	59	19
46220198	[335, 336, 338, 339]	All	735	305	385	60	739	301	381	56
46220203	[335, 336, 338, 339]	All	735	305	385	60	739	301	381	56
46220206	[335, 336, 338, 339]	All	735	305	385	60	739	301	381	56
47010184	[1452, 1453, 1454, 1455]	All	180	60	80	25	176	64	84	29
Benson No. 35, Rural municip...	[335, 336, 338, 339]	M(+)	240	50	140	35	236	54	144	39
47010167	[335, 336, 338, 339]	M(+)	240	50	140	35	236	54	144	39
Stockholm, Village (VL)	[350, 351, 353, 354]	F(+)	170	50	90	45	174	46	86	41
47050250	[350, 351, 353, 354]	F(+)	170	50	90	45	174	46	86	41
47060449	[1452, 1453, 1454, 1455]	All	170	35	100	20	166	39	104	24
Outlook, Town (T)	[1452, 1453, 1454, 1455]	All	1005	575	385	30	1001	579	389	34
47110515	[1665, 1666, 1667, 1668]	F(+)	220	55	15	135	216	59	19	139
47110562	[96, 97, 98, 99]	All	125	70	35	35	129	66	31	31
48062654	[1452, 1453, 1454, 1455]	All	195	80	115	15	199	76	111	11
48060157	[350, 351, 353, 354]	F(+)	265	60	170	50	269	56	166	46
48060309	[350, 351, 353, 354]	M(+)	325	35	245	60	329	31	241	56
48060906	[350, 351, 353, 354]	F(+)	210	25	155	45	214	21	151	41
48060907	[350, 351, 353, 354]	F(+)	210	25	155	45	214	21	151	41
48061157	[335, 336, 338, 339]	F(+)	210	35	140	20	206	39	144	24
48062209	[350, 351, 353, 354]	F(+)	350	50	225	60	346	54	229	64
48100333	[335, 336, 338, 339]	All	3180	995	1975	225	3184	991	1971	221
48111668	[350, 351, 353, 354]	F(+)	255	60	160	50	259	56	156	46
48110648	[96, 97, 98, 99]	All	85	30	30	40	89	26	26	36
48110850	[1452, 1453, 1454, 1455]	All	390	215	170	20	394	211	166	16
48112132	[335, 336, 338, 339]	M(+)	350	100	185	50	346	104	189	54
48112197	[335, 336, 338, 339]	M(+)	505	145	325	20	501	149	329	24
Division No. 13, Census div...	[1404, 1405, 1406, 1407]	M(+)	3560	1335	2195	15	3556	1339	2199	19
48130225	[350, 351, 353, 354]	F(+)	205	40	135	45	209	36	131	41
24030051	[335, 336, 338, 339]	All	865	145	555	150	861	149	559	154
24170032	[350, 351, 353, 354]	F(+)	205	25	105	60	201	29	109	64
Québec, Territoire équivalent (TÉ)	[492, 493, 494, 495]	F(+)	435	360	65	25	439	356	61	21
24230153	[90, 91, 92, 93]	F(+)	120	50	15	40	116	54	19	44
24230942	[350, 351, 353, 354]	All	400	80	275	30	396	84	279	34
24231102	[90, 91, 92, 93]	F(+)	205	115	20	55	201	119	24	59
Saint-Janvier-de-Joly, Municip...	[1665, 1666, 1667, 1668]	F(+)	525	15	15	510	529	11	11	506
24360139	[335, 336, 338, 339]	F(+)	240	40	140	75	244	36	136	71
24420077	[340, 341, 343, 344]	All	145	20	60	50	141	24	64	54
24430228	[350, 351, 353, 354]	All	505	115	255	150	509	111	251	146
24470165	[340, 341, 343, 344]	All	95	25	40	15	91	29	44	19
24470207	[350, 351, 353, 354]	All	445	90	245	125	449	86	241	121
24490187	[335, 336, 338, 339]	F(+)	505	150	320	50	509	146	316	46
24490191	[340, 341, 343, 344]	F(+)	35	15	20	15	39	11	16	11
24490301	[1665, 1666, 1667, 1668]	All	435	20	30	400	439	16	26	396
Nicolet, Ville (V)	[350, 351, 353, 354]	F(+)	3995	730	2285	995	3999	726	2281	991
24530139	[335, 336, 338, 339]	M(+)	205	40	125	55	209	36	121	51
24530093	[335, 336, 338, 339]	M(+)	225	35	130	75	229	31	126	71

24530126	[335, 336, 338, 339]	M(+)	220	20	95	120	224	16	91	116
24580034	[350, 351, 353, 354]	All	530	95	330	120	534	91	326	116
24580511	[335, 336, 338, 339]	F(+)	395	70	205	105	391	74	209	109
24600077	[350, 351, 353, 354]	All	415	55	265	110	419	51	261	106
24640015	[335, 336, 338, 339]	M(+)	460	110	250	85	456	114	254	89
24650073	[350, 351, 353, 354]	M(+)	295	50	185	45	291	54	189	49
24650511	[340, 341, 343, 344]	All	185	35	95	40	181	39	99	44
24650634	[1452, 1453, 1454, 1455]	All	445	125	305	30	449	121	301	26
24660814	[350, 351, 353, 354]	F(+)	285	65	175	30	281	69	179	34
24661010	[335, 336, 338, 339]	F(+)	310	45	185	65	306	49	189	69
24661488	[335, 336, 338, 339]	F(+)	240	35	150	40	236	39	154	44
24661853	[340, 341, 343, 344]	All	220	25	175	35	224	21	171	31
24662824	[335, 336, 338, 339]	M(+)	220	45	125	35	216	49	129	39
24662925	[350, 351, 353, 354]	All	550	125	365	75	554	121	361	71
24662984	[335, 336, 338, 339]	F(+)	325	75	205	60	329	71	201	56
24662985	[335, 336, 338, 339]	F(+)	325	75	205	60	329	71	201	56
24663196	[335, 336, 338, 339]	All	405	65	280	75	409	61	276	71
24660853	[335, 336, 338, 339]	M(+)	280	40	140	115	284	36	136	111
Roussillon, Municipalité ...	[2054, 2055, 2056, 2057]	F(+)	13960	20	725	13230	13964	16	721	13226
24670100	[350, 351, 353, 354]	F(+)	510	130	315	80	514	126	311	76
24710265	[335, 336, 338, 339]	F(+)	375	115	215	30	371	119	219	34
24730094	[335, 336, 338, 339]	M(+)	210	45	140	40	214	41	136	36
24750209	[335, 336, 338, 339]	M(+)	730	180	470	95	734	176	466	91
24790111	[335, 336, 338, 339]	All	575	115	345	100	571	119	349	104
24810194	[335, 336, 338, 339]	F(+)	390	40	170	165	386	44	174	169
24810319	[1665, 1666, 1667, 1668]	All	1170	190	150	845	1174	186	146	841
24820095	[335, 336, 338, 339]	F(+)	440	100	280	75	444	96	276	71
24930178	[350, 351, 353, 354]	M(+)	655	130	375	165	659	126	371	161

Table 11. Categories Explored for Invariant-Free Exact Inference. Note that we choose to include some attributes that only represent 25% of the data; these pose little threat because they are already subsampled from the population, but they are interesting targets nonetheless.

Parent Attribute	Sub #1	Sub #2	Sub #3	Sub #4
Age				
29: 85 years and over	85 to 89 years	90 to 94 years	95 to 99 years	100 years and over
Marital Status				
61: Living common-law	Living common law ...	Living common law ...	Living common law ...	Living common law ...
66: Not married and not ...	Not married and ...	Not married and ...	Not married and ...	Not married and not ...
Family Size				
71: Total - Census families in private ...	2 persons	3 persons	4 persons	5 or more persons
Household Income				
276: \$100,000 and over	100,000 to 124,999	125,000 to 149,999	150,000 to 199,999	\$200,000 and over
Official Language				
388: Total - First official language spoken ...	English	French	English and French	Neither English nor French
Mother Tongue, Single Response				
402: Cree-Innu languages	Atikamekw	Cree languages	Innu (Montagnais)	Naskapi
420: Ojibway languages	Anishinaabemwin (Chippewa)	Daawaamwin (Ojawa)	Saulteau (Western Ojibway)	Ojibway, n.o.s.
457: Iroquoian languages	Cayuga	Mohawk	Oneida	Iroquoian languages, n.i.e.
473: Siouan languages	Assiniboine	Dakota	Stoney	Siouan languages, n.i.e.
500: Cushitic languages	Bilen	Oromo	Somali	Cushitic languages, n.i.e.
572: Serbo-Croatian	Bosnian	Croatian	Serbian	Serbo-Croatian, n.i.e.
581: Celtic languages	Irish	Scottish Gaelic	Welsh	Celtic languages, n.i.e.
588: High German languages	German	Pennsylvania German	Swiss German	Yiddish
600: Scandinavian languages	Danish	Icelandic	Norwegian	Swedish
674: Nilo-Saharan languages	Dinka	Nuer	Nilo-Saharan languages, n.i.e.	African, n.o.s.
Major Field of Study Age 15+ (25%)				
2064: Mathematics, computer ...	11. Computer and inf...	25. Library science	27. Mathematics and statistics	30D Interdisciplinary math...
Major Field of Study Age 25-64+ (25%)				

2127: Mathematics, computer ...	11. Computer and inf...	25. Library science	27. Mathematics and statistics	30D Interdisciplinary math...
Workplace Misc (25%)				
2593: Total - Place of work ...	Worked at home	Worked outside Canada	No fixed workplace address	Usual place of work
2598: Total - Commuting destination ...	Commute within census ...	Commute to a ...	Commute to a ...	Commute to a ...

Table 12. Categories Explored for Invariant-Free Probabilistic Inference. Note that we choose to include some attributes that only represent 25% of the data; these pose little threat because they are already subsampled from the population, but they are interesting targets nonetheless.

Parent Attribute	Sub #1	Sub #2	Sub #3
Age			
10: 0 to 4 years	5 to 9 years	10 to 14 years	15 to 64 years
Family			
89: Total - Persons in census families	Married spouses or common-law partn	Parents in one-parent families	Children
96: Total - Persons not in census famil	Living alone	Living with other relatives	Living with non-relatives only
Income			
297: \$100,000 and over	100,000to124,999	125,000to149,999	\$150,000 and over
335: Total - LIM low-income status in 20	0 to 17 years	18 to 64 years	65 years and over
340: In low income based on the Low-inco	0 to 17 years	18 to 64 years	65 years and over
350: Total - LICO low-income status in 2	0 to 17 years	18 to 64 years	65 years and over
355: In low income based on the Low-inco	0 to 17 years	18 to 64 years	65 years and over
Mother Tongue, Single Response			
417: Ojibway-Potawatomi languages	Anicinabemowin (Algonquin)	Oji-Cree	Ojibway languages
432: Slavey-Hare languages	Deh Gah Ghotie Zhatie (South Slavey)	Satuotine Yati (North Slavey)	Slavey, n.o.s.
443: Tutchone languages	Northern Tutchone	Southern Tutchone	Tutchone, n.o.s.
478: Tsimshian languages	Gitksan (Gitksan)	Nisga'a	Tsimshian
492: Berber languages	Kabyle	Tamazight	Berber languages, n.i.e.
508: Aramaic languages	Assyrian Neo-Aramaic	Chaldean Neo-Aramaic	Aramaic, n.o.s.
517: Austro-Asiatic languages	Khmer (Cambodian)	Vietnamese	Austro-Asiatic languages, n.i.e
632: Persian languages	Dari	Iranian Persian	Persian (Farsi), n.o.s.
679: Sign languages	American Sign Language	Quebec Sign Language	Sign languages, n.i.e.
702: Tai-Kadai languages	Lao	Thai	Tai-Kadai languages, n.i.e.
713: Uralic languages	Estonian	Finnish	Hungarian
Indigenous (25%)			
1404: Single Indigenous responses	First Nations (North American India	Métis	Inuk (Inuit)
1452: Total - Private households by numbe	One-maintainer household	Two-maintainer household	Three-or-more-maintainer household
1502: Single Indigenous ancestry (only)	First Nations (North American India	Métis single ancestry	Inuit single ancestry
1512: Single Indigenous and non-Indigenou	First Nations (North American India	Métis and non-Indigenous ancestry o	Inuit and non-Indigenous ancestry o
Generation Status (25%)			
1665: Total - Generation status for the p	First generation	Second generation	Third generation or more
Education (25%)			
2002: Postsecondary certificate or diplom	Apprenticeship or trades certificat	College, CEGEP or other non-univers	University certificate or diploma b
2018: Postsecondary certificate or diplom	Apprenticeship or trades certificat	College, CEGEP or other non-univers	University certificate or diploma b
2054: Business, management and public adm	30.16 Accounting and computer scien	44. Public administration and socia	52. Business, management, marketing
2117: Business, management and public adm	30.16 Accounting and computer scien	44. Public administration and socia	52. Business, management, marketing
Location of Study (25%)			
2219: Oceania	Australia	New Zealand	Other locations of study in Oceania
Language used Most Often at Work (25%)			
2317: Ojibway-Potawatomi languages	Anicinabemowin (Algonquin)	Oji-Cree	Ojibway languages
2332: Slavey-Hare languages	Deh Gah Ghotie Zhatie (South Slavey)	Satuotine Yati (North Slavey)	Slavey, n.o.s.
2341: Tutchone languages	Northern Tutchone	Southern Tutchone	Tutchone, n.o.s.
2349: Inuktit (Inuit) languages	Inuinnaqtun (Inuvialuktun)	Inuktitut	Inuktit (Inuit) languages, n.i.e.
2355: Iroquoian languages	Mohawk	Oneida	Iroquoian languages, n.i.e.
2375: Tsimshian languages	Gitksan (Gitksan)	Nisga'a	Tsimshian
2389: Berber languages	Kabyle	Tamazight	Berber languages, n.i.e.
2403: Aramaic languages	Assyrian Neo-Aramaic	Chaldean Neo-Aramaic	Aramaic, n.o.s.
2410: Austro-Asiatic languages	Khmer (Cambodian)	Vietnamese	Austro-Asiatic languages, n.i.e.
2503: Persian languages	Dari	Iranian Persian	Persian (Farsi), n.o.s.
2538: Sign languages	American Sign Language	Quebec Sign Language	Sign languages, n.i.e.
2560: Tai-Kadai languages	Lao	Thai	Tai-Kadai languages, n.i.e.
2570: Uralic languages	Estonian	Finnish	Hungarian